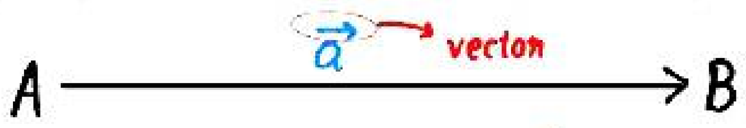
Vector Algebra

✓ **Vector** : A quantity that has magnitude as well as direction is called a vector. denoted by $\overrightarrow{AB}$ or $\vec{a}$.



✓ **Initial point** : The point A where from the vector $\overrightarrow{AB}$ starts is known as initial point.

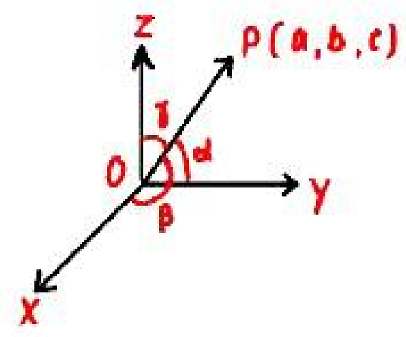
✓ **Terminal point** : The point B, where it ends is said to be the terminal point.

✓ **Magnitude** : The distance between initial and terminal points of a vector is called the magnitude (or length) of the vector.

✓ **Scalar** : Those physical quantities which have only magnitude are called scalar, e.g., area, volume, mass etc.

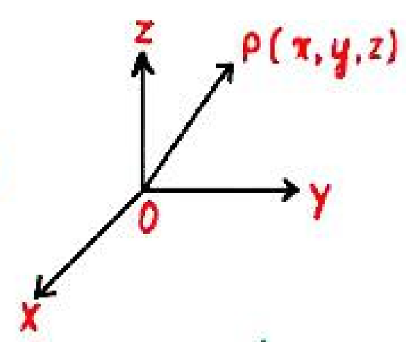
✓ **Direction cosines** : If $\vec{r} = a\hat{i} + b\hat{j} + c\hat{k}$ makes angle α, β, γ with +ve direction of x-axis, y-axis and z-axis respectively, then $\cos\alpha$, $\cos\beta$ and $\cos\gamma$ are the direction cosines of $\vec{r}$ and are denoted by l, m and n where,

$\boxed{l = \cos\alpha = \frac{a}{\sqrt{a^2+b^2+c^2}}}$, $\boxed{m = \cos\beta = \frac{b}{\sqrt{a^2+b^2+c^2}}}$, $\boxed{n = \cos\gamma = \frac{c}{\sqrt{a^2+b^2+c^2}}}$



✓ **Direction ratios** : If numbers a, b, c are propotional to direction cosines l, m and n respectively of $\vec{r}$, then a, b, c are called direction ratios of $\vec{r}$.

✓ **Position vector** : Consider a point (x, y, z) in space. The vector $\overrightarrow{OP}$ with initial point, origin O and terminal point P, is called the position vector of P.



✓ **zero vector** : A vector whose initial and terminal points coincide is known as zero vector.

✓ **Unit vector** : A vector whose magnitude is unity is said to be unit vector. denoted by $\hat{a}$. $\boxed{\hat{a} = \frac{1}{|\vec{a}|}\vec{a}}$

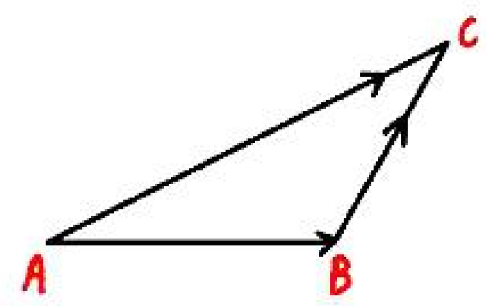
✓ **Co-initial Vectors** : Two or more vectors having the same initial point are called coinitial vectors.

✓ **Collinear Vectors** : Two or more vectors are said to be collinear if they are parallel to the same line, irrespective of their magnitudes and directions.

✓ **Equal vectors** : Two vectors $\vec{a}$ and $\vec{b}$ are said to be equal, if they have the same magnitude and direction regardless of the positions of their initial points, and written as $\vec{a} = \vec{b}$

✓ **Negative of a vector** : A vector whose magnitude is the same as that of a given vector, but direction is opposite to that of it, is called negative of the given vector. $\boxed{\overrightarrow{BA} = -\overrightarrow{AB}}$

✓ **Addition of vectors** : $\boxed{\overrightarrow{AC} = \overrightarrow{AB} + \overrightarrow{BC}}$ (Triangle law of vector addition)



✓ **Properties of vector addition** : (i) $\boxed{\vec{a}+\vec{b} = \vec{b}+\vec{a}}$ (ii) $\boxed{(\vec{a}+\vec{b})+\vec{c} = \vec{a}+(\vec{b}+\vec{c})}$

(iii) $\boxed{\vec{a}+\vec{0} = \vec{0}+\vec{a} = \vec{a}}$

✓ **Multiplication of a vector by a scalar** : $\boxed{|\lambda\vec{a}| = |\lambda||\vec{a}|}$ scalar

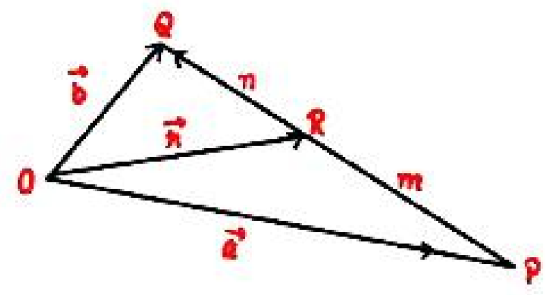
✓ **Midpoint** $\boxed{\vec{r} = \frac{\vec{a}+\vec{b}}{2}}$

Note : For any scalar k, $\boxed{k\vec{0} = \vec{0}}$

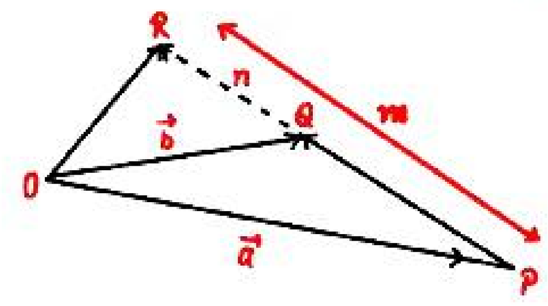
✓ **Component form** : $\boxed{\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}}$ vector component $\boxed{|\vec{r}| = \sqrt{x^2+y^2+z^2}}$ x, y, z = scalar components of $\vec{r}$

✓ **vector joining two points** : $\boxed{|\overrightarrow{P_1P_2}| = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}}$

✓ **Section formula** : Case I When R divides PQ internally $\boxed{\vec{r} = \dfrac{m\vec{b}+n\vec{a}}{m+n}}$



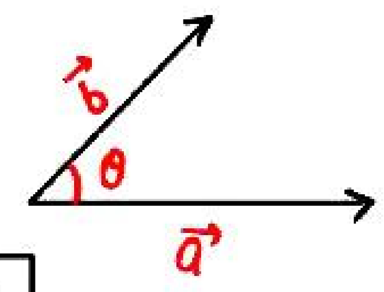
Case II When R divides PQ externally $\boxed{\vec{r} = \dfrac{m\vec{b}-n\vec{a}}{m-n}}$



✓ **Scalar (or dot) product of two vectors** :

$\boxed{\vec{a}\cdot\vec{b} = |\vec{a}||\vec{b}|\cos\theta}$ $\boxed{\cos\theta = \dfrac{\vec{a}\cdot\vec{b}}{|\vec{a}||\vec{b}|}}$ $\boxed{\theta = \cos^{-1}\left[\dfrac{\vec{a}\cdot\vec{b}}{|\vec{a}||\vec{b}|}\right]}$

θ is the angle between $\vec{a}$ and $\vec{b}$, $0 \le \theta \le \pi$



✓ **Properties** :

(1) $\boxed{\vec{a}\cdot\vec{b} = \vec{b}\cdot\vec{a}}$

(2) $\boxed{\vec{a}\cdot(\vec{b}+\vec{c}) = \vec{a}\cdot\vec{b}+\vec{a}\cdot\vec{c}}$

(3) $\boxed{(\lambda\vec{a})\cdot\vec{b} = \lambda(\vec{a}\cdot\vec{b}) = \vec{a}\cdot(\lambda\vec{b})}$

(4) $\boxed{\vec{a}\cdot\vec{b} = 0 \Leftrightarrow \vec{a}=0,\ \vec{b}=0 \text{ OR } \vec{a}\perp\vec{b}}$

(5) If $\vec{a} = a_1\hat{i}+a_2\hat{j}+a_3\hat{k}$ and $\vec{b} = b_1\hat{i}+b_2\hat{j}+b_3\hat{k}$, then $\vec{a}\cdot\vec{b} = a_1b_1+a_2b_2+a_3b_3$

(6) Projection of $\vec{a}$ on $\vec{b}$ = $\dfrac{\vec{a}\cdot\vec{b}}{|\vec{b}|}$ and position vector of $\vec{a}$ on $\vec{b}$ $\left|\dfrac{\vec{a}\cdot\vec{b}}{|\vec{b}|^2}\right|\cdot\vec{b}$

(7) Projection of $\vec{b}$ on $\vec{a}$ = $\dfrac{\vec{a}\cdot\vec{b}}{|\vec{a}|}$ and position vector of $\vec{b}$ on $\vec{a}$ $\left|\dfrac{\vec{a}\cdot\vec{b}}{|\vec{a}|^2}\right|\cdot\vec{a}$

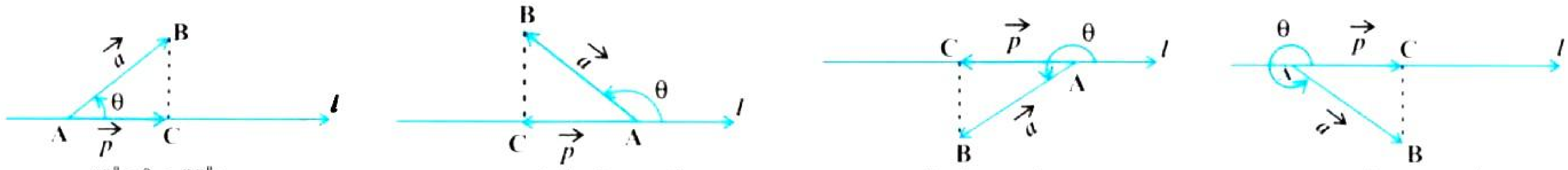
Note : If two vectors $\vec{a}$ and $\vec{b}$ are given in component form as $a_1\hat{i}+a_2\hat{j}+a_3\hat{k}$ and $b_1\hat{i}+b_2\hat{j}+b_3\hat{k}$

$\boxed{\vec{a}\cdot\vec{b} = a_1b_1+a_2b_2+a_3b_3}$

✓ **Observations** :

1. $\vec{a}\cdot\vec{b}$ is a real number.
2. Let $\vec{a}$ and $\vec{b}$ be two non-zero vectors, then $\vec{a}\cdot\vec{b}=0$ if and only if $\vec{a}$ and $\vec{b}$ are perpendicular to each other i.e. $\boxed{\vec{a}\cdot\vec{b}=0 \Leftrightarrow \vec{a}\perp\vec{b}}$
3. If $\theta=0$ then $\vec{a}\cdot\vec{b} = |\vec{a}||\vec{b}|$. In particular $\vec{a}\cdot\vec{a} = |\vec{a}|^2$, as θ in this case is 0.
4. If $\theta=\pi$ then $\vec{a}\cdot\vec{b} = -|\vec{a}||\vec{b}|$. In particular $\vec{a}\cdot(-\vec{a}) = -|\vec{a}|^2$, as θ in this case is π.
5. In view of the Observations 2 and 3, for mutually perpendicular unit vectors $\hat{i},\hat{j}$ and $\hat{k}$, we have $\boxed{\hat{i}\cdot\hat{i}=\hat{j}\cdot\hat{j}=\hat{k}\cdot\hat{k}=1}$ $\boxed{\hat{i}\cdot\hat{j}=\hat{j}\cdot\hat{k}=\hat{k}\cdot\hat{i}=0}$
6. The angle between two non-zero vectors $\vec{a}$ and $\vec{b}$ is given by $\boxed{\cos\theta = \dfrac{\vec{a}\cdot\vec{b}}{|\vec{a}||\vec{b}|}}$
7. The scalar product is commutative i.e $\boxed{\vec{a}\cdot\vec{b} = \vec{b}\cdot\vec{a}}$

✓ **Projection of a vector on a line** : The $\vec{p}$ is called the projection vector and its magnitude $|\vec{p}|$ is simply called as the projection of the vector $\overrightarrow{AB}$ on the directed line l.



(0° < θ < 90°) (i) (90° < θ < 180°) (ii) (180° < θ < 270°) (iii) (270° < θ < 360°) (iv)

✓ **Observations** :

1. If $\vec{p}$ is the unit vector along a line l, then the projection of a vector $\vec{a}$ on the line l is given by $\vec{a}\cdot\hat{p}$.

2. Projection of a vector $\vec{a}$ on other vector $\vec{b}$, is given by $\boxed{\vec{a}\cdot\vec{b} \text{ OR } \vec{a}\cdot\left[\frac{\vec{b}}{|\vec{b}|}\right] \text{ OR } \frac{1}{|\vec{b}|}(\vec{a}\cdot\vec{b})}$

3. If $\theta = 0$, then the projection vector of $\overrightarrow{AB}$ will be $\overrightarrow{AB}$ itself and if $\theta = \pi$, then the projection vector of $\overrightarrow{AB}$ will be $\overrightarrow{BA}$.

4. If $\theta = \frac{\pi}{2}$ or $\theta = \frac{3\pi}{2}$, then the projection vector of $\overrightarrow{AB}$ will be zero vector.

Note : If α, β and γ are the direction angles of vector $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$, then its direction cosines may be given as

$$\boxed{\cos\alpha = \frac{\vec{a}\cdot\hat{i}}{|\vec{a}||\hat{i}|} = \frac{a_1}{|\vec{a}|}}, \quad \boxed{\cos\beta = \frac{a_2}{|\vec{a}|}} \quad \text{and} \quad \boxed{\cos\gamma = \frac{a_3}{|\vec{a}|}}$$

a_1, a_2, a_3 = scalar components

$\vec{a}$ = unit vector

$$\boxed{\vec{a} = \cos\alpha\,\hat{i} + \cos\beta\,\hat{j} + \cos\gamma\,\hat{k}}$$

Vector (or cross) product of two vectors

$$\boxed{\vec{a}\times\vec{b} = |\vec{a}||\vec{b}|\sin\theta\,\hat{n}} \quad \text{OR} \quad \boxed{\sin\theta = \frac{|\vec{a}\times\vec{b}|}{|\vec{a}||\vec{b}|}}$$

θ is the angle between $\vec{a}$ and $\vec{b}$, $0 \le \theta \le \pi$

$\hat{n}$ = unit vector perpendicular to the plane $\vec{a}$ and $\vec{b}$

Properties :

(1) $\boxed{\vec{a}\times\vec{b} = -\vec{b}\times\vec{a}}$

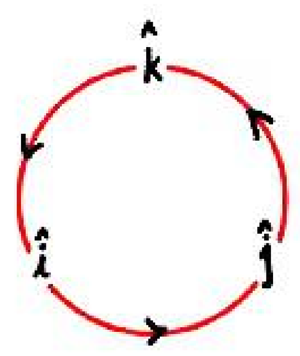
(2) $\boxed{\text{If } \vec{a}\times\vec{b} = 0 \Leftrightarrow \vec{a} = 0,\ \vec{b} = 0 \text{ or } \vec{a} \parallel \vec{b}}$

(3) If $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ and $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$, $\vec{a}\times\vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$

Observations :

1. $\vec{a}\times\vec{b}$ is a vector.
2. If $\theta = \frac{\pi}{2}$ then $\boxed{\vec{a}\times\vec{b} = |\vec{a}||\vec{b}|}$
3. Angle between two vectors $\vec{a}$ and $\vec{b}$ $\boxed{\sin\theta = \frac{|\vec{a}\times\vec{b}|}{|\vec{a}||\vec{b}|}}$
4. $\boxed{\hat{i}\times\hat{i} = \hat{j}\times\hat{j} = \hat{k}\times\hat{k} = \vec{0}}$

   $\boxed{\hat{i}\times\hat{j} = \hat{k},\ \hat{j}\times\hat{k} = \hat{i},\ \hat{k}\times\hat{i} = \hat{j}}$
5. $\boxed{\hat{j}\times\hat{i} = -\hat{k}}$, $\boxed{\hat{k}\times\hat{j} = -\hat{i}}$ and $\boxed{\hat{i}\times\hat{k} = -\hat{j}}$



Area of triangle ABC $= \frac{1}{2}|\vec{b}||\vec{a}|\sin\theta = \boxed{\frac{1}{2}|\vec{a}\times\vec{b}|}$

Area of parallelogram ABCD $= |\vec{b}||\vec{a}|\sin\theta = \boxed{|\vec{a}\times\vec{b}|}$

Projection formulae :

(1) $\boxed{a = b\cos C + c\cos B}$ (2) $\boxed{b = c\cos A + a\cos C}$ (3) $\boxed{c = a\cos C + b\cos a}$